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Enriques and octonionic magic supergravity models

Massimo Bianchi[ab] **and Sergio Ferrara**[acd]

[a] *Physics Department, Theory Unit, CERN, CH 1211, Geneva 23, Switzerland*
[b] *Dipartimento di Fisica, Universitá di Roma "Tor Vergata" and INFN Sezione di Roma "Tor Vergata", Via della Ricerca Scientifica, 00133 Roma, Italy*
[c] *INFN - Laboratori Nazionali di Frascati, Via Enrico Fermi 40, 00044 Frascati, Italy*
[d] *Department of Physics and Astronomy, University of California, 90095-1547, Los Angeles, CA U.S.A.*
E-mail: massimo.bianchi@roma2.infn.it, Sergio.Ferrara@cern.ch

Abstract: We reconsider the Enriques Calabi Yau (FHSV) model and its string derivation and argue that the Octonionic magic supergravity theory admits a string interpretation closely related to the Enriques model. The uplift to $D = 6$ of the Octonionic magic model has 16 abelian vectors related to the rank of Type I and Heterotic strings.

Keywords: D-branes, F-Theory, Black Holes in String Theory, String Duality.

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1. Introduction

Among the magical supergravities [1], related to the famous magic square of Freudenthal, Rozenfeld and Tits of the division algebras R, C, H, O, there is just one, the Octonionic model with 27 vector multiplets and 28 hypermultiplets, which cannot be obtained as a truncation of $\mathcal{N}=8$ supergravity. This is obviously due to the fact that such theory is based on the real form $E_{7(-25)}$ of the exceptional group E_7 while $\mathcal{N}=8$ supergravity is based on the real form $E_{7(+7)}$. As a consequence, the corresponding moduli space of the $\mathcal{N}=2$ and $\mathcal{N}=8$ supergravities based on $E_{7(-25)}$ (Octonions) and $E_{7(+7)}$ (split Octonions) are very different. The former is the rank 3, 54-dimensional Kähler space $E_{7(-25)}/E_6 \times \mathrm{U}(1)$. The latter is the rank 7, 70-dimensional non-Kähler space $E_{7(7)}/\,\mathrm{SU}(8)$. However, following Gunaydin et al. [1] the $\mathcal{N}=2$ $E_{7(-25)}$ model is completely unified since the 28 vectors (27 matter vectors and one graviphoton) mix under $E_{7(-25)}$ electric-magnetic duality rotations. This symmetry is not a symmetry of the Lagrangian but only of the field equations. The maximal symmetry of the Lagrangian, which does not mix electric and magnetic potentials, being $SU^*(8)$ [2]. The possible relation with the FHSV model [3] comes from the observation that $E_{7(-25)}$ (unlike $E_{7(7)}$!) contains as maximal subgroup $\mathrm{SO}(2,10)\times\mathrm{SU}(1,1)$ and indeed, for the FHSV model, the space

$$\frac{\mathrm{SO}(2,10)}{\mathrm{SO}(2)\times\mathrm{SO}(10)}\times\frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)} \tag{1.1}$$

is the moduli space of complex structure deformations of the underlying space which is a torus fibration of an Enriques surface $CY_{\rm FHSV} \approx \mathcal{E} \times T^2$ with holonomy $\mathrm{SU}(2) \times Z_2$. Indeed we have

$$\frac{\mathrm{SO}(2,10)}{\mathrm{SO}(2)\times\mathrm{SO}(10)} \times \frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)} \subset \frac{E_{7(-25)}}{E_6\times\mathrm{U}(1)} \tag{1.2}$$

Note that this moduli space cannot be obtained as a truncation of $E_{7(+7)}/\mathrm{SU}(8)$, the moduli space of $\mathcal{N}=8$ supergravity.

In an analogous way the hypermultiplet moduli space, that includes deformations of the Kähler structure of $CY_{\rm FHSV} \approx \mathcal{E} \times T^2$, is obtained by c-map [4] to be

$$\frac{\mathrm{SO}(12,4)}{\mathrm{SO}(12)\times\mathrm{SO}(4)} \tag{1.3}$$

and is a quaternionic subspace of the exceptional quaternionic manifold obtained by c-map from the Octonionic magic model [4]

$$\frac{\mathrm{SO}(12,4)}{\mathrm{SO}(12)\times\mathrm{SO}(4)} \subset \frac{E_{8(-24)}}{E_7\times\mathrm{SU}(2)} \tag{1.4}$$

Note that the moduli space cosets of the Octonionic model and of the FHSV model have the same rank (respectively 3 and 4 for the special and quaternionic manifolds). The number of vector multiplets as well as hypermultiplets is augmented by 16 each with respect to the FHSV model

$$n_V^{\mathbf{O}} = 27 = 11+16 \quad , \quad n_H^{\mathbf{O}} = 28 = 12+16 \tag{1.5}$$

and quite remarkably 16 is the rank of the gauge group in Type I and Heterotic models in $D=10$.

Both models correspond to self-mirror CY threefolds with $h_{11}=h_{21}=11$ and $h_{11}=h_{21}=27$, respectively, and admit an uplift to $D=6$ with $\mathcal{N}=(1,0)$ supersymmetry. The $D=6$ interpretation is in terms of $n_T=9$ tensor multiplets, $n_H=12$ hypermultiplets and $n_V=0$ vector multiplets for the FHSV model and $n_T=9$ tensor multiplets, $n_H=28$ hypermultiplets and $n_V=16$ vector multiplets for the Octonionic model.

We will give a simple construction of the two models and show analogies and differences. Electric-magnetic duality in $D=4$ and special geometry are discussed in section 2. In section 3, we discuss BPS and non BPS black-holes solutions and attractors in the two magic models. In section 4 we describe the embedding of the parent $D=6$ models in Type I superstring and F-theory on Voisin-Borcea (VB) orbifolds. Our concluding remarks and comments on other magic models are in section 5. An appendix contains details of the construction of the Type I superstring models underlying the two magic models.

2. Duality rotations

In this section we discuss the duality properties [5] of the effective $\mathcal{N}=2$ theory for the Enriques CY and the Octonionic magic model. As we have seen before, there is a common sector of the two models which comes from the $\mathcal{N}=(1,0)$ tensor multiplets after Kaluza

Klein reduction from $D = 6$. This gives rise to the vector multiplet (from the tensor multiplets plus KK vectors) moduli space[1]

$$\frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)} \times \frac{\mathrm{SO}(2,10)}{\mathrm{SO}(2)\times\mathrm{SO}(10)} \tag{2.1}$$

The cubic holomorphic polynomial, from which the $\mathcal{N}=2$ prepotential of the underlying special geometry for the FHSV model arises, is

$$F_{\mathrm{FHSV}}(X) = \frac{s}{2}\eta_{IJ}x^I x^J \tag{2.2}$$

where $\eta_{IJ}x^I x^J = x_{10}^2 - \sum_{i=1}^{9} x_i^2$. Note that $F(X)$ has manifest $\mathrm{SO}(1,9)$ invariance because of the Lorentzian contraction of the 10 X coordinates. These originate from the nine tensor scalars related to the classical moduli space of those K3 moduli which survive on the Enriques surface. Therefore $\mathrm{SO}(1,9)$ does not give electric-magnetic duality transformations since it does not mix the electric vectors with their duals. The moduli corresponding to ReX are the axions that come from the two forms and have an associated shift-symmetry. The larger symmetry $\mathrm{SO}(2,10)$ does mix electric and magnetic field strengths, contrary to the Heterotic string where an analogous symmetry, $\mathrm{SO}(2,n_V)$ T-duality, does not act as electric-magnetic duality rotations.

Let us now move to the Octonionic model with $h_{11} = h_{21} = 27$. In this case the cubic polynomial, as it would come from a six-dimensional interpretation, is [7, 8]

$$F_{\mathrm{OM}}(X) = \frac{s}{2}\left(\eta_{IJ}x^I x^J\right) - x_I C^I_{ab} v^a v^b \tag{2.3}$$

where $\eta_{IJ} = (1,-1,-1,\ldots,-1)$ and v^a $(a = 1,\ldots 16)$ are the complex scalars in the $\mathcal{N}=2$ vector multiplets, that can be identified with the 6-D vector fields of the Cartan subalgebra along the two compactified directions. The structure constants C^I_{ab} determine the coupling of (tensor multiplet) scalars to the 16 vectors in $D = 6$ in the Cartan subalgebra of $\mathrm{U}(16)$. Note that C^I_{ab} satisfy the cocycle condition [8–11]

$$\eta_{IJ} C^I_{(ab} C^J_{cd)} = 0 \tag{2.4}$$

which follows from gauge invariance of the six dimensional theory in the Coulomb phase where $\mathrm{U}(16)$ is Higgsed to $\mathrm{U}(1)^{16}$ and massive states are integrated out. If we demand that each choice in the complex structure of the Enriques surface be $\mathrm{SO}(1,9)/\mathrm{SO}(9)$ equivalent, then C^I_{ab} must be the (symmetric) γ matrices of the $\mathrm{SO}(1,9)$ Clifford algebra, being the 16 vectors a chiral $\mathrm{SO}(1,9)$ spinor representation, which is real and inequivalent to $16'$. Still, the $\mathrm{SO}(1,9)$ symmetry is not an e.m. duality in 4D since it does not mix electric with magnetic field strengths. However when $\mathrm{SO}(1,9) \to \mathrm{SO}(2,10)$ an $\mathrm{SO}(2,10)$ chiral spinor representation has 32 real components and indeed in this case the action of $\mathrm{SO}(2,10)$ mixes the 4D vectors with their duals. This phenomenon is similar to the action of T-duality on R-R fields in type II superstring theory.[2] We also remark that the requirement that

[1] This is the space L(8,0)=L(0,8) in the notation of [6], while the Octonionic model is L(8,1).

[2] Indeed $\mathrm{SO}(6,6)$ and $\mathrm{SO}(2,10)$ are two inequivalent real forms of $\mathrm{SO}(12)$.

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each point on the $\mathrm{SO}(1,9)/\mathrm{SO}(9)$ moduli space gives equivalent Physics is the key to the enlargement of the manifest $\mathrm{SO}(2,10)\times\mathrm{SU}(1,1)$ to the exceptional group $E_{7(-25)}$. Indeed, under $\mathrm{SO}(2,10)$ the $12+16=28$ vectors, together with their duals, form a 56 dimensional space as follows $56=(12,2)+(32,1)$. This is identical to the decomposition which takes place in type II supergravity if one decomposes $E_{7(7)}$ with respect to the T-duality sub group $\mathrm{SO}(6,6)$ and the axion-dilaton symmetry $\mathrm{SL}(2,R)$

$$56=(12_{NS-NS},2)+(32_{R-R},1) \tag{2.5}$$

The above consideration explains our previous remark.

3. Extreme black-holes and attractors

The two models under consideration have also interesting properties as far as extreme black-hole solutions are concerned. Since their moduli spaces fall in the classification of symmetric spaces in the literature [12], we just comment on their attractor solutions. Both models have both BPS and non BPS black-holes depending on which orbit the charge vector (24 dimensional in the first case, 56 dimensional in the second case) lies in. The classification of orbits for the FHSV model yields [12]

$$\begin{array}{cc} BPS & \dfrac{\mathrm{SU}(1,1)\times\mathrm{SO}(2,10)}{\mathrm{SO}(2)\times\mathrm{SO}(10)} \\ NBPS\ (Z\neq 0) & \dfrac{\mathrm{SU}(1,1)\times\mathrm{SO}(2,10)}{\mathrm{SO}(1,1)\times\mathrm{SO}(1,9)} \\ NBPS\ (Z=0) & \dfrac{\mathrm{SU}(1,1)\times\mathrm{SO}(2,10)}{\mathrm{SO}(2)\times\mathrm{SO}(2,8)} \end{array} \tag{3.1}$$

The 11 complex moduli are all fixed in the BPS orbit while there is a moduli space in the NBPS case [13] (the $\mathcal{N}=2$ central charge Z is a section of the Kähler $\mathrm{U}(1)$ bundle)

$$\begin{array}{cc} NBPS\ (Z\neq 0) & \mathrm{SO}(1,9)/\mathrm{SO}(9)\times\mathrm{SO}(1,1) \\ NBPS\ (Z=0) & \mathrm{SO}(2,8)/\mathrm{SO}(2)\times\mathrm{SO}(8) \end{array} \tag{3.2}$$

The previous considerations exhaust the analysis of the FHVS model.

For the Octonionic theory the classification of attractors is as follows, the charge orbits are [12]

$$\begin{array}{cc} BPS & \dfrac{E_{7(-25)}}{E_6} \\ NBPS\ (Z\neq 0) & \dfrac{E_{7(-25)}}{E_{6(-26)}} \\ NBPS\ (Z=0) & \dfrac{E_{7(-25)}}{E_{6(-14)}} \end{array} \tag{3.3}$$

The residual moduli space of the non BPS attractors are [13]

$$NBPS\ (Z\neq 0) \qquad \frac{E_{6(-26)}}{F_4}$$

$$NBPS\ (Z=0) \qquad \frac{E_{6(-14)}}{SO(2)\times \mathrm{SO}(10)} \tag{3.4}$$

Note that all moduli spaces of all non BPS orbits of the FHVS and Octonionic magic model have in common the restricted moduli space $\mathrm{SO}(1,8)/\mathrm{SO}(8)$. This is the tensor multiplet moduli space of non BPS self-dual string for 9 tensor multiplets, one of the tensor moduli being fixed by the six-dimensional version of the attractor mechanism [14 − 16].

For all these models, the classical 4D black-hole entropy of the attractor solutions is given by the following formula [15, 17]

$$S = \pi\sqrt{|\mathcal{I}_4|} \tag{3.5}$$

where $\mathcal{I}_4$ is an electric-magnetic duality invariant combination of the electric and magnetic charges of the theory. For the FHSV model $\mathcal{I}_4$ is the unique singlet in the product of four $(2,12)$ irreps of $\mathrm{SL}(2)\times\mathrm{SO}(2,10)$. For the Octonionic model $\mathcal{I}_4$ is the unique singlet in the product of four 56 irreps of $E_{7(-25)}$.

4. Model building

As previously observed, the two $\mathcal{N}=2$ supergravity models can be obtained from compactification on T^2 of $\mathcal{N}=(1,0)$ chiral supergravity models in $D=6$. Both of them have the same number of tensor multiplets, $n_T=9$. While the parent of the FHSV has $n_H=12$ hypermultiplets and $n_V=0$ vector multiplets, the parent of the Octonionic magic model has $n_H=28$ and $n_V=16$.

Models of this kind can be embedded in string theory. The most efficient way is to consider unoriented descendants of Type IIB superstrings on $K3$ [18, 19] which in many cases can be related to F-theory compactifications on elliptically fibered CY spaces [20, 21] with constant dilaton [22]. Perturbative Heterotic models can only have one tensor multiplet in their massless spectrum and are thus unsuitable for our purposes.[3] For certain choices of the compactification, including the choice of the internal gauge bundle, heterotic $\mathcal{N}=(1,0)$ models in $D=6$ can be related to F-theory or to Type I. In particular the compactification on T^4/Z_2 with gauge group $\mathrm{U}(16)$ [23] can be shown to be dual to a Type I compactification found in [19] and recently discussed in [24] as a playground for non-perturbative effects.

Focussing on unoriented descendants of the Type IIB superstring in $D=6$ [18, 19, 25 − 31], one starts with $\mathcal{N}=(2,0)$ supergravity coupled to 21 tensor multiplets, each containing an anti self-dual tensor and 5 scalars. The moduli space is $\mathrm{SO}(5,21)/\mathrm{SO}(5)\times\mathrm{SO}(21)$. The unoriented worldsheet parity projection of the closed string spectrum, coded in the Klein bottle amplitude, produces $\mathcal{N}=(1,0)$ supergravity coupled to $n_T^{\rm cl}$ tensor multiplets and $n_H^{\rm cl}$ 'neutral' hypermultiplets. Since both kinds of matter multiplets descend from the 21 $\mathcal{N}=(2,0)$ tensor multiplets, of the parent Type IIB theory, one has a constraint

$$n_T^{\rm cl} + n_H^{\rm cl} = 21 \tag{4.1}$$

[3]Including NS5-branes may lead to models with several tensor multiplets that however lack a full-fledged string description. In some cases these models can be related to M-theory compactifications on $K3\times S^1/Z_2$ with M5-branes, supporting tensor multiplets

Explicit constructions have produced models with $n_T^{\rm cl}$ ranging from 0 [25] to 19 [20 – 22, 30, 31]. Since the open string spectrum cannot produce any massless tensor multiplet, it is impossible to exceed $n_T = 19$ in this kind of models.

Chiral anomaly cancellation in $D = 6$ [32], which is equivalent to R-R tadpole cancellation in this kind of compactifications [33, 34], puts severe constraints on the massless spectrum. In particular absence of gravitational anomalies requires

$$29 n_T + n_H - n_V = 273 \tag{4.2}$$

Notice that (anti) self-dual antisymmetric tensor do contribute to the anomaly. Tensorini and hyperini have, say, L chirality while gaugini and gravitini have R chirality and contribute with the opposite sign to the anomaly. One can immediately conclude that $n_T = 9$ is the maximum value for a theory without vector multiplets and indeed a theory with $n_V = 0$, $n_T = 9$ and $n_H = 12$ is completely free from anomaly in the sense that the anomaly polynomial is exactly zero as for the parent Type IIB theory. In fact the latter is exactly twice the former (2 gravitini, 2×21 L fermions and $5 - 21$ tensors). As we will see, this model corresponds to a Type I theory without open strings [28, 25] or to F-theory on a Voisin - Borcea (VB) orbifold that makes use of the freely-acting Enriques involution of $K3$.

When vector multiplets are present, the irreducible gauge anomaly is proportional to the quartic Casimir and cancels only for a very restricted class of models, i.e. the ones for which

$$Tr_{\rm Adj} F^4 = \sum_H Tr_{R_H} F^4 \tag{4.3}$$

In Type I models vector and charged hypermultiplets correspond to open string excitations of various D-branes present in the background and coded in the Annulus amplitude and its Möbius strip projection. In perturbative models, open strings have only two ends and they can at most tranform in the product of two fundamental representations of classical groups.

Once (4.3), (4.2) are satisfied, one can invoke a generalization of the G-S mechanism to cancel the left-over reducible gauge, gravitational and mixed anomalies [35, 32, 36]. Actually there are two kinds of mechanisms. The first one involves (anti) self-dual antisymmetric tensors and serves to cancel anomalies of the form

$$I_{4+4} = \frac{1}{2} \sum_I X_4^I \wedge X_4^I \tag{4.4}$$

where $I = 0, 1, \ldots n_T$ for the mechanism to work at all. The second one involves 4-forms dual to axions [23, 37] and serves to cancel anomalies of the form[4]

$$I_{2+6} = \sum_h X_2^h \wedge X_6^h \tag{4.5}$$

where $h = 1, \ldots n_H$ for the mechanism to work at all. In string theory modular invariance and tadpole cancellation guarantee the necessary couplings [33]

$$L_{\rm GSS} = \sum_I C_2^I \wedge X_4^I + \sum_h \left[C_4^h \wedge X_2^h + C_0^h X_6^h \right] \tag{4.6}$$

[4] Four-dimensional remnants of anomaly cancellation are the generalized Chern-Simons couplings discussed in [38 – 41].

Terms of the form $C_4 \wedge X_2 \equiv C_4 \wedge TrF$ can be dualized to $*dC_0 \wedge A = A^\mu \partial_\mu C_0$. The field C_0 is a Stückelberg field for A or in other words the (abelian) gauge field A gauges the axionic shift symmetry and becomes massive. The mechanism can take place in a supersymmetric fashion and lifts entire hypermultiplets.

In relation to gauge anomaly cancellation, the antisymmetric combinations of two 8-dimensional representations such as the 28 of SO(8) (Adjoint) or the 28 and 28* of U(8) as well as the 27 of Sp(8) (the antisymmetric singlet decouples) play a peculiar role. Their contribution to the irreducible anomaly vanishes and they can thus appear in arbitrary number in the spectrum. Indeed in F-theory compactifications on VB orbifolds, that we will momentarily review briefly, the singularities of the fibration are of D_4 type and give rise to products of SO(8) gauge groups and hypers in the Adjoint.

Before doing that, let us briefly recall some aspects of the low-energy effective action which are relevant for our analysis. First of all $\mathcal{N} = (1,0)$ supersymmetry in $D = 6$, very much like $\mathcal{N} = 2$ supersymmetry in $D = 4$, prevents neutral coupling of hypers to vectors. As a consequence the gauge coupling can only depend on the real scalars in the tensor multiplets [7, 9–11]. For perturbative heterotic string compactifications, the only such scalar is the dilaton and the dependence is linear (tree level) plus a constant (one loop GS counterterm). In Type I models or F-theory compactifications at constant (perturbative thus vanishingly small) coupling, the parity even counterpart of the GSS counterterm, dictated by supersymmetry, reads

$$L_{\text{kin}} = \sum_I v_I C^I_{ab} Tr \left(F^a F^b \right) \tag{4.7}$$

where v_I is an $\mathrm{SO}(1, n_T)$ vector and C^I_{ab} is a set of $n_T + 1$ structure constants satisfying

$$\eta_{IJ} C^I_{(ab} C^J_{cd)} = \sum_f Tr_{R_f} \left(T_a T_b T_c T_d \right) \tag{4.8}$$

for anomaly cancellation i.e. gauge invariance of the one-loop effective lagrangian. It is clear that a combination satisfying

$$\eta_{IJ} C^I_{(ab} C^J_{cd)} = 0 \tag{4.9}$$

is gauge invariant per se and is thus not related to one-loop anomaly cancellation and can always be present even in the absence of chiral fermions. Notice that contrary to heterotic models the Type I dilaton lies in a hypermultiplet and does not play a role in this context [23, 24]. In fact we have already mentioned that it is possible to construct Type I and F-theory models with $n_T = 0$ [25] whose (non-perturbative) heterotic dual would exist only at a fixed value for the dilaton. As already observed, the tensor scalars moduli space is

$$\mathrm{SO}(1, n_T)/\,\mathrm{SO}(n_T) \tag{4.10}$$

A large class of tractable models is given by F-theory compactifications on VB orbifolds. These are elliptically fibered CY threefolds with a base of the form $B = K3/\sigma$ with σ an antiholomorphic involution of $K3$ that reverses the holomorphic 2-form $\sigma\omega_{2,0} = -\omega_{2,0}$.

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The resulting CY is given by $X = K3 \times T^2/\sigma'$ where σ' combines σ with the Z_2 action $Z \to -Z$ on the torus coordinate. As a result the holomorphic 3-form $\omega_{3,0} = \omega_{2,0} \wedge dZ$ is invariant. The classification due to Nikulin is given in terms of three integers (r, a, δ) with $\delta = 0, 2$ representing the 'parity' of the canonical class, $1 \leq r \leq 20$ the rank of the σ-invariant sublattice of $H^2(K3, Z)$ and $1 \leq a \leq 11$ the rank of the Picard lattice of $K3/\sigma$. For $(r, a) \neq (10, 10), (10, 8)$, the Hodge numbers of the base $B = K3/\sigma$ and the threefold $X = K3 \times T^2/\sigma'$ are given by

$$h_{11}(B) = r \quad , \quad h_{11}(X) = 5 + 3r - 2a \quad , \quad h_{11}(X) = 65 - 3r - 2a \tag{4.11}$$

Moreover the elliptic fibration degenerates at $k = (r-a)/2$ rational curves (spheres) E_i and at a curve of genus $g = (22-r-a)/2$. The degenerations are all of the D_4 type, equivalent to 4 D7-branes on an $\Omega 7^-$-plane, i.e. a bound state of 7-branes with no monodromy and thus constant dilaton. The resulting gauge group is $\mathrm{SO}(8)^{k+1}$ with g hypers in the Adjoint of the SO(8) gauge group associated to the curve of genus g. Notice that for sufficiently high g the latter can completely Higgs this factor but the remaining k are always unbroken. It is an easy exercise to compute and factorize the anomaly polynomial

$$I_{\mathrm{FTonVB}} = 2\sum_{i=1}^{k}(X_i - Y)^2 + [(r-10) + (a-10)](X_0 - Y) \tag{4.12}$$

where

$$Y = \frac{1}{32\pi^2} trR^2 \quad , \quad X_0 = \frac{1}{8\pi^2} trF_0^2 \quad , \quad X_i = \frac{1}{8\pi^2} trF_i^2 \tag{4.13}$$

with 0 labelling the group associated to the genus g curve. It is also easy to check that the number of tensors, which is r after inclusion of the self-dual one in the supergravity multiplet, is always larger than $k + 1$, the number of terms in the reducible anomaly polynomial. One can expect the GSS mechanism to be at work. Notice that the case (10,10,0) is special and corresponds to the Enriques involution which has no fixed points where the torus fibration could degenerate. The anomaly polynomial is exactly zero (since $n_T = 9$ and $n_H = 12$ and $n_V = 0$) and does not require any GS-like mechanism. The elliptic threefold is the one considered by FHSV that has $h_{11}(X) = h_{21}(X) = 11$. One might be tempted to associate the octonionic magic model to the VB orbifold (10,4,0) with gauge group $\mathrm{SO}(8)^4$ of rank 16. However this cannot work in $D = 6$ since one of the SO(8) factor is singlet out wrt to the other three. The three adjoint hypers can fully break the former while the latter three remain unbroken. After compactification to $D = 4$, one can turn on VEV's for the complex scalars in the vector multiplets and go to the Coulomb phase where the gauge group is broken to its maximal torus (Cartan) and all the charged hypers can get a mass. This indeed gives a CY threefold compactification with $h_{11} = h_{21} = 27$ and the correct number of vector and hyper multiplets in $D = 4$ dimensions. Yet it is difficult to envisage a restoration of a full symmetry among the 16 Cartan vectors. We believe the correct 6-D description of the Octonionic magic model requires a different construction in terms of Type I to which we now turn.

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4.1 The octonionic model

A possible candidate for a 6-D parent of the Octonionic magic model is a Type I compactification on T^4/Z_2 with a peculiar unoriented projection. In the untwisted sector one can combine Ω with an order two shift à la Scherk-Schwarz in any of the internal coordinates [28, 27, 25, 30, 31]. Although the massless spectrum, carrying zero KK momentum, is completely unaffected and gives rise to $\mathcal{N} = (1, 0)$ supergravity coupled to one tensor and 4 neutral hyper multiplets, the transverse channel amplitude exposing massless RR tadpole gets crucially modified in that no massless RR tadpole associated to Ω9-planes is present. The correct interpretation, possibly after T-duality, is that one is superimposing an equal number of two mutually supersymmetric Ω-planes with opposite R-R charge [42–45]. Since the geometric SS shift does nothing to the winding states that get projected by the Klein bottle one has still 16 Ω5-planes that carry non-vanishing RR charge. In the twisted sector this exotic Ω projection keeps 8 tensor multiplets and 8 hypermultiplets. In all one has $n_T^{\rm cl} = 9$ tensor and $n_H^{\rm cl} = 12$ neutral hyper multiplets. Although the field content is non anomalous there is still an untwisted RR tadpole (not associated to chiral anomalies [33, 34]) to be cancelled. It requires the introduction of 16 dynamical D5-brane ad their unoriented open string excitations. The absence of twisted R-R tadpoles, consequent to the choice of splitting 16 fixed points into 8 (hypers) and 8 (tensors), implies that the Chan-Paton embedding of the Z_2 should be freely acting and leads to a $\mathrm{U}(16)$ group coupled to one hypermultiplet in the Adjoint representation. Further details can be found in the appendix. The anomaly polynomial is once again exactly zero and one can go to the Coulomb branch where $\mathrm{U}(16) \to \mathrm{U}(1)^{16}$. The amusing feature of this breaking pattern is that the 16 vectors are all on the same footing as required for their being part of an irreducible representation such as the spinor of SO(1, 9). In the Coulomb branch the gauge couplings are given by

$$v_I C^I_{ab} \tag{4.14}$$

where C^I_{ab} are the symmetric γ matrices of SO(1, 9) that satisfy the cocycle condition as required by gauge invariance. It is amusing to observe that precisely the same cocycle condition allows a Fierz rearrangement that is necessary in order to prove supersymmetry of the vector multiplet Lagrangian in $D = 10$!

4.2 The Enriques FHSV model

The Enriques FHSV model can be constructed similarly. One starts, for instance, with a Type IIB compactification on T^4/Z_2 and performs a Klein bottle projection that combines Ω with a Z_2 involution of T^4/Z_2 without fixed points [28, 27, 25, 30, 31, 42–45]. This is nothing but the Enriques involution at a sublocus of the moduli space where $K3 \approx T^4/Z_2$.

Contrary to the previous case the resulting unoriented closed string model is not only anomaly free, in the sense that the anomaly polynomial exactly vanishes, but also free from R-R tadpoles in the transverse channel. This prevents the possibility of introducing D-brane and their open string excitations altogether. As a consequence the Type I model has $n_T^{\rm cl} = 9$, $n_H^{\rm cl} = 12$ and $n_V = 0$, that is what is needed to produce the FHSV model after compactification on a T^2.

Alternatively one can construct an equivalent model as an asymmetric orbifolds of Type IIB. Indeed S-duality of Type IIB in $D = 10$ relates the symmetry Ω (worldsheet parity) to $(-)^{F_L}$ (change of sign of all R-R fields). Although quotienting (i.e. 'gauging') Ω and $(-)^{F_L}$ gives different results in $D = 10$, i.e. Type I in the former case and Type IIA in the latter, combining Ω and $(-)^{F_L}$ with an order two involution of a compactification leads to equivalent models in lower dimensions [46 – 48]. For our purposes one can check that quotienting Type IIB on $K3$ by $(-)^{F_L}\sigma_{\mathcal{E}}$ yields an anomaly free $\mathcal{N} = (1,0)$ model with $n_T = 9$, $n_H = 12$ and $n_V = 0$. In particular one can perform the analysis at a point in the $K3$ moduli space where $K3 \approx T^4/Z_2$ such as in fermionic constructions [49 – 53, 55] or in Gepner models [25, 56, 57].

4.3 Other magic models

By using asymmetric orbifolds and free fermion constructions [49 – 53] Kounnas et al [54] have been able to construct magic hyper-free $\mathcal{N} = 2$ supergravities in $D = 4$. We would like to comment on the possibility of constructing other $D = 6$ models which can play the role of parents for the magic $\mathcal{N} = 2$ supergravities with $n_V = 8+5+2 = 15$, $n_V = 4+3+2 = 9$ and $n_V = 2+2+2 = 6$, that enjoy SO(1, 5), SO(1, 3) and SO(1, 2) symmetry respectively since the $D = 6$ vector multiplets in the Coulomb phase (after Higgsing) transform as spinors of dimension 8, 4 and 2 respectively. Once again it is amusing to observe that these are precisely the dimensions and spinor representations that allow consistent supersymmetric Yang-Mills Lagrangian. The cocycle conditions on the structure constants that determine the coupling of the scalars in tensor multiplets to the vector fields are reinterpreted as the possibility of performing a the necessary Fierz rearrangement on four Fermi terms that appear after varying the gauge fields.

Many $\mathcal{N} = (1,0)$ superstring models with $n_T = 5$ and $n_H^{\rm cl} = 16$ are known [18, 19, 25] with rank higher than 8. For our purposes, a particularly interesting class are models where charged hypers transform in the 28-dimensional adjoint of $\mathrm{SO}(8)^2$ or the 28-dimensional of U(8) or the 27-dimensional of $\mathrm{Sp}(8)^2$. The pattern of symmetry breaking in all these cases yields $\mathrm{U}(1)^8$ with neutral hypers. Gravitational anomaly cancellation fixes the number of neutral hypers once the number of tensor and vector multiplets is fixed. The former by the choice of unoriented closed string projection and the latter by the choice of gauge symmetry breaking pattern which is tantamount to the choice of Wilson lines on D9's and position of D5's. The models labelled by D_{16}, A_{64} in [25] can accomplish the task. Also F-theory on the VB orbifold (6,4,0) with $\mathrm{SO}(8)^2$ gauge group and $g = 6$ hypers in the (1, 28) representation could do the job after compactification to $D = 4$. However, as for the (10,4,0) case, it is hard to envisage the origin of the SO(1, 5) symmetry among the vectors in $D = 6$.

Fewer models with $\mathcal{N} = (1,0)$ superstring models with $n_T = 3$ or $n_T = 2$ are known. In order to get $n_V = 4$ or $n_V = 2$ neutral vector multiplets coupled to neutral hypers one has to start with models with at least U(4) or SO(8) or Sp(8) for the former or SU(2) (which is GS cancellable, lacking a quartic Casimir!). There are choices that however do not seem to yield the desired pattern of symmetry breaking. Once again F-theory on VB orbifolds with $r = 4$ and $r = 3$ respectively and $a = 4$ or $a = 2$ and $a = 1$ respectively could do the job after compactification on T^2 but obscure the origin of the $\mathrm{SO}(1, n_T)$ symmetry among the massless vectors in the Coulomb phase in $D = 6$. For $n_T = 2$, one can perform a different unoriented projection of the unique Type I model with $n_T = 0$ in $D = 6$, based on the $(k = 1)^6$ Gepner model [25], and keep $n_T = 2$. Stringent constraints from tadpole cancellation seem however to naively prevent this possibility.

It is not clear that magic supergravity models in different dimensions have a unique embedding in superstring constructions.

5. Further comments and conclusions

Our analysis so far has been essentially classical. Quantum corrections may a priori spoil the beautiful geometry of the two magic models under consideration. However it has been known for a while that perturbative and non-perturbative corrections to the 2-derivative effective action vanish in the FHSV Enriques model [3]. The argument is based on heterotic / Type II duality [58]. The hypermultiplet geometry is exact in the heterotic description since the dilaton belongs in a vector multiplet. The special geometry is exact in the type IIB description, since the dilaton belongs in a hypermultiplet and no worldsheet instantons are present since the Enriques CY threefold is self-mirror. The same sort of argument applies to the quaternionic magic model. As we will momentarily observe, the moduli space is a fibration over the moduli space of the FHSV Enriques model which is uncorrected as we have just seen. Moreover the massless open string spectrum, consisting in the Coulomb phase of 16 neutral vector multiplets and as many hyper multiplets, enjoys $\mathcal{N} = 4$ supersymmetry and has thus zero β-function and produces no corrections to the two-derivative effective action. Yet there may be interesting threshold corrections to four and higher derivative terms in the effective action such as the ones computed in [59] for the FHSV Enriques model. For related work on BPS states in the FHSV model see [60, 61].

Before concluding, we would like to comment on the two possible Higgs mechanisms mentioned in the paper. Notice that a long vector multiplet (16 states: 8 bosons and 8 fermions) in a 4D sense corresponds to nonzero VEV for hyper-scalars and zero VEV for vector-scalars. A short vector multiplet instead (8 states: 4 bosons and 4 fermions) corresponds to zero VEV for hyper-scalars and non-zero VEV for vector-scalars. Obviously only the former admits a 6D uplift since there are no BPS particle (point-like) states in 6D $\mathcal{N} = (1, 0)$ supersymmetric theories.

We would also like to comment on the decomposition of the magic moduli spaces as fibrations

$$M_q = B_q + F_q \tag{5.1}$$

In (Type I) string theory the base B_q should describe closed string moduli, while the fiber F_q describes open string moduli. It is amusing to observe that the fiber precisely matches (at least for $D = 4, 5$) the moduli space of non-BPS attractor solutions [13]. In all there are 12 models forming three sequences of four exceptional geometries, associated to the four division algebras $J_3^R, J_3^C, J_3^H, J_3^O$. They correspond to $D = 5, 4, 3$ dimensions and $q = 1, 2, 4, 8$, one has

$$dimM_q = 3q + (7 - D) \quad , \quad dimB_q = q + (7 - D) \quad , \quad dimM_q = 2q \quad , \tag{5.2}$$

q	Scalar Manifold M_q	Base B_q	Fiber F_q
8	$\frac{E_{6(-26)}}{F_4}$	$\frac{\mathrm{SO}(9,1)}{\mathrm{SO}(9)} \times \mathrm{SO}(1,1)$	$\frac{F_{4(-20)}}{\mathrm{SO}(9)}$
4	$\frac{\mathrm{SU}^*(6)}{\mathrm{Usp}(6)}$	$\frac{\mathrm{SO}(5,1)}{\mathrm{SO}(5)} \times \mathrm{SO}(1,1)$	$\frac{\mathrm{Usp}(4,2)}{\mathrm{Usp}(4)\times\mathrm{Usp}(2)}$
2	$\frac{\mathrm{SL}(3,C)}{\mathrm{SU}(3)}$	$\frac{\mathrm{SO}(3,1)}{\mathrm{SO}(3)} \times \mathrm{SO}(1,1)$	$\frac{\mathrm{SU}(2,1)}{\mathrm{SU}(2)\times\mathrm{U}(1)}$
1	$\frac{\mathrm{SL}(3,R)}{\mathrm{SO}(3)}$	$\frac{\mathrm{SO}(2,1)}{\mathrm{SO}(2)} \times \mathrm{SO}(1,1)$	$\frac{\mathrm{SL}(2,R)}{\mathrm{SO}(2)}$

Table 1: I sequence ($D = 5$).

q	Scalar Manifold M_q	Base B_q	Fiber F_q
8	$\frac{E_{7(-25)}}{E_6\times\mathrm{U}(1)}$	$\frac{\mathrm{SO}(10,2)}{\mathrm{SO}(10)\times\mathrm{SO}(2)} \times \frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)}$	$\frac{E_{6(-14)}}{\mathrm{SO}(10)\times\mathrm{U}(1)}$
4	$\frac{SO^*(12)}{\mathrm{U}(6)}$	$\frac{\mathrm{SO}(6,2)}{\mathrm{SO}(6)\times\mathrm{SO}(2)} \times \frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)}$	$\frac{\mathrm{SU}(4,2)}{\mathrm{SU}(4)\times\mathrm{SU}(2)\times\mathrm{U}(1)}$
2	$\frac{\mathrm{SU}(3,3)}{\mathrm{SU}(3)\times\mathrm{SU}(3)\times\mathrm{U}(1)}$	$\frac{\mathrm{SU}(2,2)}{\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{U}(1)} \times \frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)}$	$\frac{\mathrm{SU}(2,1)}{\mathrm{U}(2)} \times \frac{\mathrm{SU}(1,2)}{\mathrm{U}(2)}$
1	$\frac{\mathrm{Sp}(6,R)}{\mathrm{U}(3)}$	$\frac{\mathrm{Sp}(4,R)}{\mathrm{U}(2)} \times \frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)}$	$\frac{\mathrm{SU}(2,1)}{\mathrm{U}(2)}$

Table 2: II sequence ($D = 4$).

q	Scalar Manifold M_q	Base B_q	Fiber F_q
8	$\frac{E_{8(-24)}}{E_7\times\mathrm{SU}(2)}$	$\frac{\mathrm{SO}(12,4)}{\mathrm{SO}(12)\times\mathrm{SO}(4)}$	$\frac{E_{7(-5)}}{\mathrm{SO}(12)\times\mathrm{SU}(2)}$
4	$\frac{E_{7(-5)}}{\mathrm{SO}(12)\times\mathrm{SU}(2)}$	$\frac{\mathrm{SO}(8,4)}{\mathrm{SO}(8)\times\mathrm{SO}(4)}$	$\frac{\mathrm{SO}(8,4)}{\mathrm{SO}(8)\times\mathrm{SO}(4)}$
2	$\frac{E_{6(+2)}}{\mathrm{SU}(6)\times\mathrm{SU}(2)}$	$\frac{\mathrm{SO}(6,4)}{\mathrm{SO}(6)\times\mathrm{SO}(4)}$	$\frac{\mathrm{SU}(4,2)}{\mathrm{SU}(4)\times\mathrm{SU}(2)\times\mathrm{U}(1)}$
1	$\frac{F_{4(+4)}}{\mathrm{Usp}(6)\times\mathrm{Usp}(2)}$	$\frac{\mathrm{SO}(5,4)}{\mathrm{SO}(5)\times\mathrm{SO}(4)}$	$\frac{\mathrm{Usp}(4,2)}{\mathrm{Usp}(4)\times\mathrm{Usp}(2)}$

Table 3: III sequence ($D = 3$).

where, depending on D, the dimensions are taken over real (R), complex (C) and quaternions (H), respectively.

The $D = 4$ (special geometries) and $D = 3$ (quaternionic geometries) cases are related to one another by c-map [4]. The decompositions are summarized in tables 1–3

Note that the third column of Sequence II has also been recently found in the framework which relates Magic Models to constrained instantons [62], while the group $E_{8(-24)}$ (first entry in Sequence III) is the exceptional group used in [63] in a (hopeless) attempt to unify gravity with the Standard Model.

Finally, we would like to comment on the 'hyper-free' magic models of Kounnas, Dolivet and Julia [54][5] based on left-right asymmetric constructions (shift orbifolds or free fermions) with $\mathcal{N} = (4, 1)$ worldsheet susy [49, 51, 52]. Their construction consists in a two-step procedure. The first step yields a model with $\mathcal{N} = 2{+}4, 2{+}2, 2{+}1$ spacetime susy. The second step breaks all susy associated to right-movers and yields $\mathcal{N} = 2+0$ spacetime susy. Differently from 'standard' compactifications with $\mathcal{N} = 1 + 1$ spacetime susy, such as CY compactifications, the axio-dilaton belongs in a vector multiplet, like in the heterotic string on $K3\times T^2$, not in the 'universal' hypermultiplet! The minimal hyper-free theory has a sin-

[5]We would like to thank B. Julia and C. Kounnas for explaining to us their construction prior to publication [64].

gle minimally coupled vector multiplet S ($K = -\log(S+\bar{S})$) associated to the axio-dilaton.

The first non-minimal magic hyper-free theory, associated to the Jordan algebra J_3^C, has 9 $\mathcal{N}=2$ vector multiplets (18 real scalars) plus one graviphoton and moduli space

$$\mathcal{M}_3 = \frac{\mathrm{SU}(3,3)}{\mathrm{SU}(3)\times\mathrm{SU}(3)\times\mathrm{U}(1)}$$

like in $\mathcal{N}=3$ supergravity with 3 vector multiplets, which therefore are in different representations of the duality group $\mathrm{SU}(3,3)$, the threefold selfdual antisymmetric (for $\mathcal{N}=2$) and the fundamental (for $\mathcal{N}=3$).

The second, associated to the Jordan algebra J_3^H, contains 15 vector multiplets (30 real scalars) plus one graviphoton. The moduli space is

$$\mathcal{M}_6 = \frac{SO^*(12)}{\mathrm{U}(6)}$$

like in $\mathcal{N}=6$ supergravity with 15 + 1 graviphotons, with identical transformation properties (32-dimensional real chiral spinor, after including the magnetic duals) under the duality group $\mathrm{SO}^*(12)$.[6]

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A. Parent type I models

In this appendix, we describe the one-loop partition functions encoding the spectra of the Type I models in $D=6$ that give rise to the two magic supergravity models in $D=4$ after compactification on T^2. In both cases, one starts from Type IIB on $T^4/Z_2 \approx K3$. In the untwisted sector, one has[7]

$$\mathcal{T}_u = \frac{1}{2}\left[\left|\sum_\alpha c_\alpha \frac{\theta_\alpha^4}{\eta^{12}}\right|^2 \Lambda_{(4,4)} + 16\left|\sum_\alpha c_\alpha \frac{\theta_\alpha^2\theta_\alpha^2(\frac{1}{2})}{\eta^6\theta_1^2(\frac{1}{2})}\right|^2\right] \tag{A.1}$$

[6]This can be taken as evidence that supersymmetric completions of theories with the same bosonic sector may differ from one another. Indeed $\mathcal{N}=2$ and $\mathcal{N}=6$ supergravities differ even at the level of the fermionic spectrum.

[7]For notational simplicity we omit the (regulated) contribution of the non-compact bosonic zero-modes and the modular integration measure.

where θ_α are Jacobi functions, η is Dedekind function, $\Lambda_{(4,4)}$ denotes the sum over generalized momenta $\vec{p}_{L/R} \approx \vec{p} \pm \vec{w}$ and c_α enforce the GSO projection. The massless spectrum consists in the $\mathcal{N} = (2,0)$ supergravity coupled to 5 tensor multiplets. In the twisted sector, one has

$$\mathcal{T}_t = \frac{16}{2}\left[\left|\sum_\alpha c_\alpha \frac{\theta_\alpha^2 \theta_\alpha^2(\frac{\tau}{2})}{\eta^6 \theta_1^2(\frac{\tau}{2})}\right|^2 + \left|\sum_\alpha c_\alpha \frac{\theta_\alpha^2 \theta_\alpha^2(\frac{1+\tau}{2})}{\eta^6 \theta_1^2(\frac{1+\tau}{2})}\right|^2\right] \tag{A.2}$$

Each of the 16 terms produces one massless $\mathcal{N} = (2,0)$ tensor multiplet. In all one thus has 21 $\mathcal{N} = (2,0)$ tensor multiplets. Each one of them decomposes into one $\mathcal{N} = (1,0)$ tensor- and one $\mathcal{N} = (1,0)$ hyper-multiplet.

For the parent of the quaternionic magic model, the Klein bottle (unoriented) projection in the untwisted sector combines world-sheet parity Ω with an order two shift $\sigma_{\vec{\delta}}$ in the internal directions

$$\mathcal{K}_u = \frac{1}{2}[P_{\vec{\delta}} + W_{\vec{0}}] \sum_\alpha c_\alpha \frac{\theta_\alpha^4}{\eta^{12}} \tag{A.3}$$

where $P_{\vec{\delta}}$ denotes the projected sum over momenta, while $W_{\vec{0}}$ denotes the (unprojected) sum over windings. As a result, the unoriented closed string spectrum at the massless level consists in $\mathcal{N} = (1,0)$ supergravity coupled to one tensor multiplet and four neutral hypermultiplets. In the twisted sector, one has

$$\mathcal{K}_t = \frac{8-8}{2} \sum_\alpha c_\alpha \frac{\theta_\alpha^2 \theta_\alpha^2(\frac{\tau}{2})}{\eta^6 \theta_1^2(\frac{\tau}{2})} \tag{A.4}$$

that yields 8 tensor multiplets and as many hypermultiplets. In all one has $n_T^{\rm cl} = 9$ and $n_H^{\rm cl} = 12$. In the transverse channel, the only massless RR tadpole comes from $\tilde{P}_{\vec{0}}$ generated by the modular S transformation of term with $W_{\vec{0}}$. In order to cancel the $\Omega 5$ tadpole, one has to introduce $N = 16$ D5-branes and their images. In the transverse channel, the Annulus and Möbius-strip amplitudes read

$$\tilde{\mathcal{A}}_{55} = \frac{1}{2\cdot 32}[2\tilde{P}_{\vec{0}} N\bar{N} + \tilde{P}_{\vec{\delta}} N^2 + \tilde{P}_{\vec{\delta}} \bar{N}] \sum_\alpha c_\alpha \frac{\theta_\alpha^4}{\eta^{12}} \tag{A.5}$$

$$\tilde{\mathcal{M}}_{5\Omega} = -\frac{2}{2}[\tilde{P}_{\vec{\delta}} N + \tilde{P}_{-\vec{\delta}} \bar{N}] \sum_\alpha c_\alpha \frac{\theta_\alpha^4}{\eta^{12}} \tag{A.6}$$

The resulting massless spectrum consists in vector and hyper multiplets in the adjoint representation of U(16). Spontaneous symmetry breaking, which is equivalent to moving the D5 branes, produces $\mathrm{U}(16) \to \mathrm{U}(1)^{16}$.

For the parent of the Enriques FHSV model, one starts with the same Type IIB compactification on T^4/Z_2 as in the previous case. In the untwisted sector, the Klein bottle projection combines world-sheet parity with an order four rotation, equivalent two an order two projection $\vec{\delta}$ on both windings and momenta

$$\mathcal{K}_u = \frac{1}{2}[P_{\vec{\delta}} + W_{\vec{\delta}}] \sum_\alpha c_\alpha \frac{\theta_\alpha^4}{\eta^{12}} \quad . \tag{A.7}$$

This has no effect on the massless states so that the untwisted unoriented closed string spectrum consists in $\mathcal{N} = (1,0)$ supergravity coupled to one tensor multiplet and four neutral hypermultiplets. In the twisted sector, one has the same Klein bottle projection $\mathcal{K}_t$ as above, that yields 8 tensor multiplets and as many hypermultiplets. In all one has $n_T^{\rm cl} = 9$ and $n_H^{\rm cl} = 12$. In the transverse channel $\mathcal{K} \to \tilde{\mathcal{K}}$ produces non massless RR tadpoles at all. As a consequence neither D9- nor D5-branes can be introduced and the model is a consistent unoriented closed string theory without open strings.

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